

# On the stock estimation for some fishery systems

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## Abstract

In this work we address the stock estimation problem for two fishery models. We show that a tool from nonlinear control theory called "observer" can be helpful to deal with the resource stock estimation in the field of renewable resource management. It is often difficult or expensive to measure all the state variables characterising the evolution of a given population system, therefore the question arises whether from the observation of certain indicators of the considered system, the whole state of the population system can be recovered or at least estimated. The goal of this paper is to show how some techniques of control theory can be applied for the approximate estimation of the unmeasurable state variables using only the observed data together with the dynamical model describing the evolution of the system. More precisely we shall consider two fishery models and we shall show how to build for each model an auxiliary dynamical system (the observer) that uses the available data (the total of caught fish) and which produces a dynamical estimation  $\hat{x}(t)$  of the unmeasurable stock state  $x(t)$ . Moreover the convergence speed of  $\hat{x}(t)$  towards  $x(t)$  can be chosen.

**Keywords:** Fishery models, Stage-structured population models, Estimation, Harvested Fish Population, Observers.

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# 1 Introduction and a short survey of *observers design*

The stock estimation is one of the most important problem in fishery science. One can quote J.A. Gulland [17]: *A major emphasis in fishery science has been on the problems of estimating current and past level using catch levels and fishing effort data.*

To make a policy decision about the exploitation of renewable resources, it is necessary to take into account the state of the resource stocks. This implies the need of a good estimate of the available resource. Mathematical models are more and more used to describe the evolution of biological systems. Here, we consider two mathematical models for fishery resources. The first one is a "stage structured" model [43, 44] that describes the dynamics of a population divided in stage-classes (according to age, length or weight) and submitted to the fishing action. The second model is a "global" model that describes the evolution of a fish population that can move between an area where it can be harvested and a reserve area where no fishing is allowed [9]. Both models are given by systems of differential equations of the form

$$\dot{x} = f(x, E), \quad (1)$$

where  $E$  is the fishing effort (it can be seen as a control or an input) and  $x(t)$  is the state of the system at time  $t$ . The state variable  $x(t)$  represents the density of the population or the number of individuals by stage. For both models, the state  $x(t)$  is not available for measurement. In practice, the only available information at time  $t$  is the value of the captures: this means that one can measure the total catch at each time  $t$ . The value of the captures can be seen as the measurable output of system (1). The output is in general a function of the state variable and the input, that is,  $y(t) = h(x(t), E)$ .

Now assuming that (1) is a "good" model of the system under consideration, if it is possible to have the value of the state at some time  $t_0$  then it is possible to compute  $x(t)$  for all  $t \geq t_0$  by integrating the differential equation with the initial condition  $x(t_0)$ . Unfortunately, it is often not possible to measure the whole state at a given time and therefore it is not possible to integrate the differential equation because one does not know an initial condition. One can only have a partial information of the state and this partial information is precisely given by  $y(t)$  the output of the system. Therefore we shall show how to use this partial information  $y(t)$  together with the given model in order to have a dynamical estimate  $\hat{x}(t)$  of the real unknown state variable  $x(t)$ . This estimate will be produced by an auxiliary dynamical system which uses the information  $y(t)$  provided by the system (1). This dynamical system is generally of the form

$$\dot{\hat{x}} = g(\hat{x}, E, y). \quad (2)$$

It can be represented by Schema 1 The estimate error is given by  $e(t) = \hat{x}(t) - x(t)$  and it satisfies the following "error equation"

$$\dot{e} = g(\hat{x}, E, y) - f(x, E) \quad (3)$$

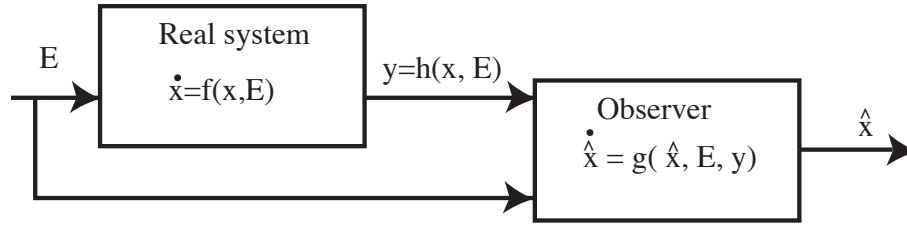


Figure 1: A schematic representation of an observer

39 The function  $g$  has to be determined in such a way that the solutions of (1) and (2)  
 40 satisfy  $x(t) - \hat{x}(t) \rightarrow 0$  as  $t \rightarrow +\infty$  regardless of the respective initial conditions of  
 41 system (1) and system (2).

42 A dynamical system (2) satisfying this conditions is called an "observer" for sys-  
 43 tem (1). When the convergence of  $\hat{x}(t)$  towards  $x(t)$  is exponential, the system (2) is  
 44 an "exponential observer". More precisely, system (2) is an exponential observer for  
 45 system (1) if there exists  $\lambda > 0$  such that, for all  $t \geq 0$  and for all initial conditions  
 46  $(x(0), \hat{x}(0))$ , the corresponding solutions of (1) and (2) satisfy

$$\|\hat{x}(t) - x(t)\| \leq \exp(-\lambda t) \|\hat{x}(0) - x(0)\|.$$

47 In this situation a good estimate of the real unmeasured state is rapidly obtained.  
 48 One must notice that we need not care about the choice of the initial condition of the  
 49 observer since the convergence of  $\hat{x}(t)$  towards the real state  $x(t)$  does not depend  
 50 on this choice.

51 When the system under consideration is a linear system, i.e., it can be written as  
 52 follows

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \\ x(t) \in \mathbb{R}^n, u(t) \in U \subset \mathbb{R}^m, y(t) \in \mathbb{R}^q, \\ A, B, \text{ and } C \text{ are respectively } n \times n, n \times m \text{ and } q \times n \text{ matrices,} \end{cases} \quad (4)$$

53 then an exponential observer (called *Luenberger Observer*)[30] for this system is  
 54 given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t)) \quad (5)$$

55 where the  $n \times q$  matrix  $K$  has to be computed. The Luenberger observer converges,  
 56 i.e.,  $|\hat{x}(k) - x(k)|$  tends to zero exponentially fast if it is possible to find a matrix  $K$   
 57 in such a way that the eigenvalues of the matrix  $A - KC$  are all with negative real  
 58 part. It has been proved that such a matrix  $K$  exists if the pair  $(C, A)$  is observable.

59 The pair  $(C, A)$  is observable if and only if the matrix:

$$O_{(C,A)} = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix}$$

60 is of rank  $n$ . In this case we say that the system (4), or the pair  $(C, A)$ , satisfies the  
61 Kalman rank condition for observability (one can see for more details and examples  
62 [39] or [20]).

63 The construction of observers for highly nonlinear systems is still a very active  
64 research area in Control Theory. Several methods have been developed for some  
65 classes of systems (one can see for instance the references [30, 27, 28, 26, 47, 13] that  
66 represent different approaches). This is not an exhaustive list, because the literature  
67 on the subject is very extensive. This active research has resulted in the emergence  
68 of many nonlinear observer design techniques. The most classical one is based on  
69 the "feedback linearization" and the observer normal form (see for instance [6], [22],  
70 [27], [46]) Roughly speaking, this method consists in finding change of coordinates  
71  $x = \kappa(z)$ ,  $u = \zeta(E)$ ,  $y = \eta(w)$  in the state space as well as in the input space and in  
72 the output space in such a way that equation (1) is transformed into

$$\begin{cases} \dot{x} = Ax + \chi(w, u), \\ w = Cx. \end{cases} \quad (6)$$

73 In this case a Luenberger type observer can be easily constructed. However the  
74 conditions under which the appropriate changes of coordinates exist are restrictive.  
75 These changes of coordinates often exist only locally and hence the derived observer  
76 design works only locally.

77 The second famous method is the high gain construction ([41],[7],[13],[14], [15], [21],  
78 [4]). A short survey is given in [4]. This method is developed hereafter and will be  
79 used in this paper.

80 Another design method uses an on-line optimization approach ([24] , [2], [33], [34],  
81 [47]) such as moving horizon observers that use the integral output prediction error  
82 in the estimation process, and the observer using Newtons method. In this case,  
83 the state is estimated by minimizing a certain norm of the difference between the  
84 ob- server output and the measured output. The advantage of the online optimiza-  
85 tion method is the capability of dealing with a variety of nonlinear systems includ-  
86 ing time-varying systems, chaotic systems, and systems with unknown parameters.  
87 Moreover this method does not require the use of any canonical form. However, the  
88 corresponding observer computations are generally quite heavy and may prevent the  
89 use of these observers for systems with very fast dynamics.

90 Historically observability theory and observers design have been developed for arti-  
91 ficial engineering systems but nowadays they are more and more applied to "natural

92 systems". We outline here some applications of nonlinear observers to biological  
93 models. Once again the list is not exhaustive.

94 In [5] the well-known Droop model which describes the growth of a population  
95 of phytoplanktonic cells is considered. Observers for this model are built and are  
96 used to discuss the validity of this model by comparing the prediction of the state  
97 computed by the observer with direct measurements of this state.

98 In [10], observers are used to estimate the kinetic rates in bioreactors. The efficiency  
99 of the observer design is illustrated with examples dealing with the microbial growth  
100 and biosynthesis reactions.

101 A robust nonlinear asymptotic observer with adjustable convergence rate has been  
102 proposed in [1]. This observer has been applied to a model of an anaerobic digestion  
103 process used for wastewater treatment.

104 The authors of [29] consider a system of populations described by the classical Lotka-  
105 Volterra model with one predator and two preys. The only available information is  
106 the total quantity of population preys without distinction between them. An ob-  
107 server is constructed that allows to estimate all the state variables. It is also shown  
108 how the observer can be used for the estimation of the level of an abiotic effect on  
109 the population system. It must be, however, noticed that the proposed observer in  
110 [29] is a local observer, i.e., its convergence is guaranteed only if the initial estimate  
111 error is small.

112 A high gain observer is used in [42] to study a system describing a one-gene regulation  
113 circuit. The observer is used to rebuild the non-measured concentrations of the  
114 mRNA and the protein.

115 The use of observer theory in fishery is scarce, we have done some works in this  
116 sense (see [35], [16]). In [35], an observer has been constructed for a stage structured  
117 discrete-time fishery model that exhibits an unknown recruitment function. In [16],  
118 a stage structured continuous model is considered and it is assumed that only the  
119 last class (mature individuals) is harvested. The present work is a continuation and  
120 a generalization of [16].

121 The goal of this paper is twofold. First we shall show that some tools from control  
122 theory are helpful to address the stock estimation problem for an exploited fish  
123 population. More precisely we shall built exponential observers for the two models  
124 under consideration. These observers will allow to give an estimate of the respective  
125 stocks. The second is to show that the application of mathematical tools to biological  
126 systems has to be done carefully. One of the most efficient way to build an observer  
127 for a nonlinear system has been given in [13]. We briefly recall the method developed  
128 in [13]. To simplify matters we consider systems without control. Roughly speaking,  
129 the result of [13] concerns systems that can be written (possibly after a coordinates

130 change):

$$\left\{ \begin{array}{l} \dot{z}(t) = \underbrace{\begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix}}_A z(t) + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \psi(z(t)) \end{pmatrix} = X(z(t)) \\ y(t) = z_1(t) = \underbrace{(1, 0, \dots, 0)}_C z(t). \end{array} \right. \quad (7)$$

131 The state of the system at time  $t$  is  $z(t) = (z_1(t), z_2(t), \dots, z_n(t)) \in \mathbb{R}^n$ , and its  
 132 measurable output is  $y(t)$ . The fact that  $y(t) = z_1(t)$  means that one can measure  
 133 only the first component of the state and hence the other components are not avail-  
 134 able for measurement. Assume that the function  $\psi$  is globally Lipschitz on  $\mathbb{R}^n$ , that  
 135 is, there exists  $K > 0$  such that  $|\psi(z) - \psi(x)| \leq K|z - x|$  for all  $(z, x) \in \mathbb{R}^n \times \mathbb{R}^n$ .  
 136 It has then been proved in [13] that for  $\theta \geq 1$  large enough, an exponential observer  
 137 (a *Luenberger type observer*) for the system (7) is given by the following dynamical  
 138 system:

$$\dot{\hat{z}} = X(\hat{z}) - S_\theta^{-1} C^T (C \hat{z} - y), \quad (8)$$

139 with  $S_\theta$  being the solution of

$$\theta S_\theta + A^T S_\theta + S_\theta A = C^T C.$$

140 System (8) is an exponential observer for system (7) means that the solutions of  
 141 (8) converge to the solutions of system (7) with an exponential speed regardless the  
 142 values of the respective initial conditions  $z(0)$  and  $\hat{z}(0)$ . To prove this result the  
 143 authors of [13] use the fact that the function  $\psi$  is globally Lipschitz on the **whole**  
 144 state space  $\mathbb{R}^n$ . The global Lipschitz assumption is very restrictive. Biological  
 145 systems always evolve in a bounded domain  $\mathcal{D}$  of  $\mathbb{R}^n$  and hence the global Lipschitz  
 146 assumption is satisfied on  $\mathcal{D}$ . However, it must be noticed that the fact that the  
 147 domain  $\mathcal{D}$  is positively invariant for system (7) and that the map  $\psi$  is globally  
 148 Lipschitz on  $\mathcal{D}$  does not guarantee the convergence of the observer (8) even if one  
 149 take the initial values inside  $\mathcal{D}$ . Indeed, the domain  $\mathcal{D}$  is positively invariant for the  
 150 system (7) but it is **not** a positively invariant set for the system (8) defining the  
 151 equations of the observer. More precisely, for a given initial condition  $(z(0), \hat{z}(0)) \in$   
 152  $\mathcal{D} \times \mathcal{D}$ , the corresponding solution  $(z(t), \hat{z}(t))$  of (7-8) can leave the set  $\mathcal{D} \times \mathcal{D}$  in  
 153 finite time: the component  $z(t)$  will actually belong to  $\mathcal{D}$  for all positive time but  
 154 there is no reason that the same property will be true for  $\hat{z}(t)$ . In order to built  
 155 an exponential observer for the considered system in this situation, one has first to  
 156 extend the function  $\psi$  from  $\mathcal{D}$  to the whole  $\mathbb{R}^n$  by a function  $\tilde{\psi}$  which is globally  
 157 Lipschitz on  $\mathbb{R}^n$  and then to consider the systems (7-8) defined on  $\mathbb{R}^n \times \mathbb{R}^n$  after  
 158 replacing the function  $\psi$  by its prolongation  $\tilde{\psi}$ . The stage-structured fishery model  
 159 we consider here will illustrate this fact. For this model, there is a domain  $\mathcal{D} \subset \mathbb{R}^3$   
 160 which is positively invariant, and the system dynamics are defined by a vector field  
 161  $X$  which is globally Lipschitz on  $\mathcal{D}$ . We shall show that the observer works well

162 when we extend the vector field  $X$  to the whole space  $\mathbb{R}^3$  and it fails to work  
 163 when the prolongation is not done. The same things are valid for the global model.  
 164 This shows that the Lipschitz extension of the vector field mentioned in [13] is not  
 165 only for mathematical sophistication purpose but it is also necessary for application  
 166 purpose. Here we construct simply a continuous Lipschitz extension of the function  
 167  $\psi$ . For more details concerning the design of Lipschitz extensions one can see for  
 168 instance [38].

169 The paper is organized as follows. In Section 2, we present the stage-structured  
 170 model and we built an observer for this system. The construction is made for a three  
 171 stages model. It can be done for an arbitrary number of stages but the calculus are  
 172 longer and more complicated. Section 3 is devoted to the stock estimation problem  
 173 for a "global" model. Once again, for clarity reasons, we have preferred to deal  
 174 with a model with two fishing areas but the observer construction can be done for a  
 175 system describing the dynamics of a fish population that can move between different  
 176 fishing zones (an example of such a system has been considered in [32]).

## 177 2 A Stage-structured model

178 In this section, we consider a class of a structured model in fishery with three classes.  
 179 The first class  $x_0$  is constitute of the pre-recruits i.e the eggs, larvae and the juveniles.  
 180 The second and the third classes are the post-recruits or the exploited phase of the  
 181 population.

182 The dynamics of the system are modeled by the following three dimensional system  
 183 (see [43, 44], [36]) :

$$\begin{cases} \dot{x}_0(t) = -\alpha_0 x_0(t) + \sum_{i=1}^2 f_i l_i x_i(t) - \sum_{i=1}^2 p_i x_i(t) x_0(t) - p_0 x_0^2(t) \\ \dot{x}_1(t) = \alpha x_0(t) - (\alpha_1 + q_1 E) x_1(t) \\ \dot{x}_2(t) = \alpha x_1(t) - (\alpha_2 + q_2 E) x_2(t) \end{cases} \quad (9)$$

184 where :

185  $x_i$ : the number of fish in the stage  $i$ .  
 186  $\alpha$  : linear aging coefficient (in time<sup>-1</sup>)  
 187  $m_i$  : natural mortality rate of class  $i$  (in time<sup>-1</sup>)  
 188  $\alpha_i = m_i + \alpha$  (in time<sup>-1</sup>)  
 189  $p_0$ : juvenile competition parameter (in time<sup>-1</sup>.number<sup>-1</sup>)  
 190  $f_i$ : fecundity rate of class  $i$  (no dimension)  
 191  $l_i$ : reproduction efficiency of class  $i$  (in time<sup>-1</sup>)  
 192  $p_i$ : predation rate of class  $i$  on class 0 (time<sup>-1</sup>.num<sup>-1</sup>)  
 193  $q_i$ : capturability coefficient of class  $i$  (in unit effort<sup>-1</sup>)  
 194  $E$ : instantaneous fishing effort. (in unit effort × time<sup>-1</sup>).

195 We assume that the total catch is available for measurement. This total catch can

196 be considered as a measurable output of the system(9) and it is given by

$$y(t) = q_1 E x_1(t) + q_2 E x_2(t) \quad (10)$$

197 We then obtain the following coupled system:

$$\begin{cases} \dot{x}_0(t) = -\alpha_0 x_0(t) + \sum_{i=1}^2 f_i l_i x_i(t) - \sum_{i=1}^2 p_i x_i(t) x_0(t) - p_0 x_0^2(t) \\ \dot{x}_1(t) = \alpha x_0(t) - (\alpha_1 + q_1 E) x_1(t) \\ \dot{x}_2(t) = \alpha x_1(t) - (\alpha_2 + q_2 E) x_2(t) \\ y(t) = q_1 E x_1(t) + q_2 E x_2(t) \end{cases} \quad (11)$$

198 We consider system (11) which is a nonlinear system. Our aim is to construct an  
199 observer (estimator) i.e an auxiliary system which will give a dynamical estimate  
200  $(\hat{x}_0(t), \hat{x}_1(t), \hat{x}_2(t))$  of the state  $(x_0(t), x_1(t), x_2(t))$  of system (9). For the construction  
201 of such auxiliary system, we shall use a method called High Gain construction (see  
202 for instance [13]). This construction provide an exponential observer; the estimation  
203 error will converges to zero with exponential speed, i.e.,

$$\|\hat{x}(t) - x(t)\| \leq \exp(-\lambda t) \|\hat{x}(0) - x(0)\|.$$

## 204 2.1 High Gain observer design for (11)

205 The system (11) is the system (9) coupled with the output (10). For the observer  
206 design, we will use the High Gain observer techniques (Gauthier et al.([13])) to  
207 construct a High Gain observer for system (9).

208 It has been proved in [43] that there is a positively invariant compact set for sys-  
209 tem (9). This set is of the form  $D = [a_0, b_0] \times [a_1, b_1] \times [a_2, b_2]$ , where the numbers  $a_i$   
210 can be chosen as small as we need and the numbers  $b_i$  are function of the parameters  
211  $f_i, l_i$  and  $p_i$ . More precisely:

$$\begin{aligned} b_i &= \pi_i \mu \\ \text{with } \pi_i &= \frac{\alpha^i}{\prod_{j=1}^i (\alpha_j + q_j E)}, \\ \text{and } \mu &= \min_{i:p_i \neq 0} \left\{ \frac{f_i l_i}{p_i} \right\} \end{aligned}$$

212 Let us denote by  $F$  the vector field defining the dynamics of the system (9), and  $h$   
213 the output function, that is  $y(t) = h(x(t)) = q_1 E x_1(t) + q_2 E x_2(t)$  and

$$214 \quad F(x(t)) = \begin{pmatrix} -\alpha_0 x_0(t) + \sum_{i=1}^2 f_i l_i x_i(t) - \sum_{i=1}^2 p_i x_i(t) x_0(t) - p_0 x_0^2(t) \\ \alpha x_0(t) - (\alpha_1 + q_1 E) x_1(t) \\ \alpha x_1(t) - (\alpha_2 + q_2 E) x_2(t) \end{pmatrix}$$

215 Let  $\Phi$  be the function  $\Phi : \overset{\circ}{D} \rightarrow \mathbb{R}^3$  ( $\overset{\circ}{D}$  is the interior of  $D$ ), defined as follows:

216  $\Phi(x) = \begin{pmatrix} h(x) \\ L_F h(x) \\ L_F^2 h(x) \end{pmatrix}$ , where  $L$  denotes the Lie derivative operator with respect to  
 217 the vector field  $F$ . Thus,

$$\Phi(x) = E \begin{pmatrix} q_1 x_1 + q_2 x_2 \\ \alpha q_1 x_0 + (\alpha q_2 - q_1(\alpha_1 + q_1 E))x_1 - q_2(\alpha_2 + q_2 E)x_2 \\ (-\alpha_0 \alpha q_1 + \alpha^2 q_2 - \alpha q_1(\alpha_1 + q_1 E))x_0 \\ + (\alpha q_1 f_1 l_1 - \alpha q_2(\alpha_1 + q_1 E) + q_1(\alpha_1 + q_1 E)^2 - \alpha q_2(\alpha_2 + q_2 E))x_1 \\ + (\alpha q_1 f_2 l_2 + q_2(\alpha_2 + q_2 E)^2)x_2 \\ - \alpha q_1 p_0 x_0^2 - \alpha q_1 p_1 x_1 x_0 - \alpha q_1 p_2 x_2 x_0 \end{pmatrix}$$

218 The Jacobian of  $\Phi$  can be written:

$$\frac{d\Phi}{dx} = \begin{pmatrix} 0 & q_1 E & q_2 E \\ \alpha q_1 E & \gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4 & \gamma_5 \end{pmatrix},$$

and

$$\left[ \frac{d\Phi}{dx} \right]^{-1} = \frac{1}{\Gamma} \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 \\ \beta_4 & \beta_5 & \beta_6 \\ \beta_7 & \beta_8 & \beta_9 \end{pmatrix},$$

219 where:

$$220 \Gamma = \text{Det} \left( \frac{d\Phi}{dx} \right) = q_1 E \gamma_2 \gamma_3 + \alpha q_1 q_2 E^2 \gamma_4 - q_2 E \gamma_1 \gamma_3 - \alpha q_1^2 E^2 \gamma_5$$

$$221 \gamma_1 = \alpha q_2 E - q_1 E(\alpha_1 + q_1 E)$$

$$222 \gamma_2 = -q_2 E(\alpha_2 + q_2 E)$$

$$223 \gamma_3 = \alpha^2 q_2 E - \alpha_0 \alpha q_1 E - \alpha q_1 E(\alpha_1 + q_1 E) - 2\alpha q_1 E p_0 x_0 - \alpha q_1 E p_1 x_1 - \alpha q_1 E p_2 x_2$$

$$224 \gamma_4 = q_1 E(\alpha_1 + q_1 E)^2 - \alpha q_2 E(\alpha_1 + q_1 E) - \alpha q_2 E(\alpha_2 + q_2 E) + \alpha q_1 f_1 l_1 E - \alpha q_1 E p_1 x_0$$

$$225 \gamma_5 = q_2 E(\alpha_2 + q_2 E)^2 + \alpha q_1 f_2 l_2 E - \alpha q_1 E p_2 x_0$$

$$226 \beta_1 = \gamma_1 \gamma_5 - \gamma_2 \gamma_4$$

$$227 \beta_2 = -q_1 E \gamma_5 + q_2 E \gamma_4$$

$$228 \beta_3 = q_1 E \gamma_2 - q_2 E \gamma_1$$

$$229 \beta_4 = -\alpha q_1 E \gamma_5 + \gamma_2 \gamma_3$$

$$230 \beta_5 = -q_2 E \gamma_3$$

$$231 \beta_6 = \alpha q_1 q_2 E^2$$

$$232 \beta_7 = \alpha q_1 E \gamma_4 - \gamma_1 \gamma_3$$

$$233 \beta_8 = q_1 E \gamma_3$$

234  $\beta_9 = -\alpha q_1^2 E^2.$

235 The determinant of  $\frac{d\Phi}{dx}$  can be written

$$\Gamma(x_0, x_1, x_2) = \text{Det}\left(\frac{d\Phi}{dx}\right) = (c + a_0x_0 + a_1x_1 + a_2x_2) E^3,$$

236 where  $c$  and  $a_i$  are functions of the parameters. The map  $(x_0, x_1, x_2) \mapsto \Gamma(x_0, x_1, x_2)$   
 237 is affine on the polyhedron  $D$ , hence it reaches its extrema on the vertexes of  $D$ . For  
 238 a given set of parameters, it is then sufficient to compute the values of  $\Gamma(x_0, x_1, x_2)$   
 239 on the vertexes of  $D$  in order to see if  $\Gamma(x_0, x_1, x_2)$  vanishes in  $D$  or not.

240 We assume that the parameters are such that the map  $\Phi$  is a diffeomorphism from  
 241  $\overset{\circ}{D}$  to  $\Phi(\overset{\circ}{D})$ . This implies that system (11) is observable.

242 In the new coordinates defined by  $(z_1, z_2, z_3)^T = z = \Phi(x) = (h(x), L_F h(x), L_F^2(x))^T$ ,  
 243 our system can be written in the canonical form as follow:

$$\begin{cases} \dot{z}(t) = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_A z(t) + \begin{pmatrix} 0 \\ 0 \\ \psi(z(t)) \end{pmatrix} \\ y(t) = z_1(t) = \underbrace{(1, 0, 0)}_C z(t). \end{cases} \quad (12)$$

244 where :  $\psi(z) = L_F^3 h(\Phi^{-1}(z)) = L_F^3 h(x) = \varphi(x)$

245 The function  $\varphi$  is smooth (it is a polynomial function of  $x = (x_0, x_1, x_2)$ ) on the  
 246 compact set  $D$ . Hence, it is globally Lipschitz on  $D$ . Therefore it can be extended  
 247 by  $\tilde{\varphi}$ , a Lipschitz function on  $\mathbb{R}^3$  which satisfies  $\tilde{\varphi}(x) = \varphi(x)$ , for all  $x \in D$ . In the  
 248 same way we define  $\tilde{\psi}$  the Lipschitz prolongation of the function  $\psi$ .

249 So we have the following system (13) defined on the whole space  $\mathbb{R}^3$ . The restriction  
 250 of (13) to the domain  $D$  is the system (12):

$$\begin{cases} \dot{z} = Az + \begin{pmatrix} 0 \\ 0 \\ \tilde{\psi}(z) \end{pmatrix}, \\ y = Cz. \end{cases} \quad (13)$$

251 Hence, we have shown that system (11) satisfies the conditions of the following result  
 252 which provides the observer construction.

253 **Proposition 2.1** ([13]) *Under the assumptions that*

254 **H1:**  $\Phi$  is a diffeomorphism from  $\overset{\circ}{D}$  to  $\Phi(\overset{\circ}{D})$ . ( $\overset{\circ}{D}$  is the interior of  $D$ ).

255 **H2:**  $\varphi$  can be extended from  $D$  to  $\mathbb{R}^3$  by a  $C^\infty$  function, globally Lipschitz on  $\mathbb{R}^3$ .

256 Then an exponential observer for system (13) is given by the following system :

$$\dot{\hat{z}} = A\hat{z} + \psi(\hat{z}) + S^{-1}(\theta)C^T(y - C\hat{z}). \quad (14)$$

257 where  $S(\theta)$  is the solution of

$$0 = -\theta S(\theta) - A^T S(\theta) - S(\theta)A^T + C^T C,$$

258 and  $\theta$  is large enough.

259 Here,  $S(\theta) = \begin{pmatrix} \theta^{-1} & -\theta^{-2} & \theta^{-3} \\ -\theta^{-2} & 2\theta^{-3} & -3\theta^{-4} \\ \theta^{-3} & -3\theta^{-4} & 6\theta^{-5} \end{pmatrix}.$

260 Precisely  $\theta \geq 2ncK\sqrt{S}$ , where  $K$  is the lipschitz coefficient of the function  $\psi$ ,  $n$  is  
261 the dimension of the space, and  $S = \sup_{i,j} |S(1)_{i,j}|$ .

262 For the proof one can see [13].

263 Going back to the our original system (9) via the transformation  $\Phi^{-1}$ , we have :

$$\dot{\hat{x}} = \tilde{F}(\hat{x}) + \left[ \frac{d\Phi}{dx} \right]_{x=\hat{x}}^{-1} \times S(\theta)^{-1}C^T(y - h(\hat{x})) \quad (15)$$

264 The restriction of this system to  $D$  is the following system :

$$\left\{ \begin{array}{l} \dot{\hat{x}}_0 = -\alpha_0 \hat{x}_0 + \sum_{i=1}^2 f_{il} \hat{x}_i - \sum_{i=1}^2 p_i \hat{x}_i \hat{x}_0 - p_0 \hat{x}_0^2 \\ \quad + (3\theta\beta_1 + 3\theta^2\beta_2 + \theta^3\beta_3)(y - q_1 E \hat{x}_1 - q_2 E \hat{x}_2) \\ \dot{\hat{x}}_1 = \alpha \hat{x}_0 - (\alpha_1 + q_1 E) \hat{x}_1 \\ \quad + (3\theta\beta_4 + 3\theta^2\beta_5 + \theta^3\beta_6)(y - q_1 E \hat{x}_1 - q_2 E \hat{x}_2) \\ \dot{\hat{x}}_2 = \alpha \hat{x}_1 - (\alpha_2 + q_2 E) \hat{x}_2 \\ \quad + (3\theta\beta_7 + 3\theta^2\beta_8 + \theta^3\beta_9)(y - q_1 E \hat{x}_1 - q_2 E \hat{x}_2) \end{array} \right. \quad (16)$$

265 which is the observer for the fishery model (9). This observer is particularly simple  
266 since it is only a copy of (9), together with a corrective term depending on  $\theta$ .

## 267 2.2 Simulations and comments

268 We present here some simulation results that show the efficiency of the observer of  
269 system (9). The simulations have been done with the free software SCILAB.

270 **Remarque 2.1** For the simulations we extend the function  $\varphi$  by continuity in order  
271 to make it globally lipschitz on  $\mathbb{R}^3$  in the following way: We denote  $\tilde{\varphi}$  the prolonga-  
272 tion of  $\varphi$  to  $\mathbb{R}^3$  and the function  $\pi$  the projection on the domain  $D$  and we construct  
273  $\tilde{\varphi} = \varphi \circ \pi$ . The extended function  $\tilde{\varphi}$  has the same Lipschitz coefficient as  $\varphi$ . The  
274 projection  $\pi$  is defined as follows: for  $x \in \mathbb{R}^3$ ,  $\pi(x) = \bar{x}$ , where  $\bar{x} \in D$  is such that  
275  $\text{dist}(x, D) = \|x - \bar{x}\|$ , i.e.,  $\bar{x}$  satisfies  $\|x - \bar{x}\| = \min_{u \in D} \|u - x\|$ . The extension algorithm  
276 is described in *Appendix B.*

277 We use the following fishery parameters [36], [43].

278  $\alpha_0 = 1.3; \alpha_1 = 0.9;$

279  $\alpha_2 = 0.85; p_0 = 0.2;$

280  $p_1 = 0.1; p_2 = 0.1;$

281  $q_1 = 0.07; q_2 = 0.15;$

282  $f_1 = 0.5; f_2 = 0.5;$

283  $l_1 = 10; l_2 = 10;$

284  $E = 0.5; \alpha = 0.8.$

285 For these parameter the Jacobian of the function  $\Phi$  is expressed as:

286

$$\frac{d\Phi}{dx} = \begin{pmatrix} 0 & 0.035 & 0.075 \\ 0.028 & 0.027275 & -0.069375 \\ -0.01458 - 0.0112x_0 & 0.0589979 - 0.0028x_0 & 0.191338 - 0.0028x_0 \\ -0.0028x_1 - 0.0028x_2 & & \end{pmatrix}$$

The determinant of this matrix is:

$$\text{Det}\left(\frac{d\Phi}{dx}\right) = 1.612 \times 10^{-6} + 0.00004697x_0 + 0.0000125265x_1 + 0.0000125265x_2.$$

287 The states  $x_0, x_1$  and  $x_2$  are time varying but remain in the positive orthant; so the  
 288  $\text{Det}\left(\frac{d\Phi}{dx}\right)$  does not vanish. Therefore  $\frac{d\Phi}{dx}$  is invertible and then  $\Phi(x)$  is a diffeomor-  
 289 phism.

290 With the parameters defined in the top of this section, we compute the coor-  
 291 dinates of the higher corner  $B$  of the parallelepiped  $D$  ([43]) and we get  $B =$   
 292  $(25; 20.639; 17.868).$

293 The nontrivial equilibrium point is  $x^* = (18.572; 15.89; 13.743).$

294 The construction of the high gain observer (15) is done with  $\theta = 17$ . For the  
 295 simulations we have taken  $x(0) = [21; 20; 15]$  and  $\hat{x}(0) = [35; 40; 10]$ .

296 **Comments:** Using the same parameters values, when we do not use the Lipschitz  
 297 prolongation of the function  $\varphi$  to the whole  $\mathbb{R}^3$ , the state estimation  $\hat{x}(t)$  computed  
 298 by the observer tends to infinity in finite time. This actually happens in the begin-  
 299 ning of the integration process as it can be seen in Figures 2, 4 and 6. When the  
 300 Lipschitz prolongation of the function  $\varphi$  to the whole  $\mathbb{R}^3$  is done, the convergence  
 301 of the estimates delivered by the observer is quite fast (Figures 3, 5 and 7).

## 302 3 A global model

### 303 3.1 The model and the observer

304 Here we consider the dynamics of a fish population moving between two zones (see  
 305 [9]). The first zone is a free fishing area, and the second zone is a reserve area

306 where no fishing is allowed. Let  $x_1(t)$  be the biomass density at time  $t$  of the fish  
 307 population in the free fishing area and  $x_2(t)$  be the biomass density at a time  $t$  of  
 308 the fish population in the reserved areas. For  $(i, j) \in \{1, 2\}^2$ , we denote by  $m_{ij}$   
 309 the migration rate from the zone  $i$  to the zone  $j$ . In the free fishing area, the total  
 310 fishing effort is denoted by  $E$ . The growth of the two sub-population in each zone  
 311 follows logistic model. The dynamics of the fish subpopulations in unreserved and  
 312 reserved areas are then assumed to be governed by the following autonomous system  
 313 of differential equations [9].

$$\begin{cases} \dot{x}_1 = r_1 x_1 \left(1 - \frac{x_1}{K_1}\right) - m_{12} x_1 + m_{21} x_2 - q E x_1 \\ \dot{x}_2 = r_2 x_2 \left(1 - \frac{x_2}{K_2}\right) + m_{12} x_1 - m_{21} x_2. \end{cases} \quad (17)$$

314  $r_1$  and  $r_2$  represent the intrinsic growth of each fish sub-population, respectively,  
 315  $K_1$  and  $K_2$  are the carrying capacities of fish species in the unreserved and reserved  
 316 areas, respectively;  $q$  is the catchability coefficient of fish species in the unreserved  
 317 area. The parameters  $r_1, r_2, q, m_{12}, m_{21}, K_1$  and  $K_2$  are positives constants.

318 To the system (17) we associate the capture (i.e. the output)  $y = q E x_1$  (the total  
 319 of caught fish in the unreserved area), with this output, we show the observability  
 320 condition of system (17) and construct an auxiliary system that will give a dynamical  
 321 estimation of the state of system (17).

322 It is possible to find a positive real number  $w_0$  in such a way that for any  $w \geq w_0$   
 323 the following compact set  $D_w$  is positively invariant for system (17). This compact  
 324 set is given

$$D_w = \{(x_1, x_2) \in \mathbb{R}_+^2 : x_1 + x_2 \leq w\},$$

325 The proof of this fact as well as the computation of  $w_0$  as a function of the parameters  
 326 are given in [Appendix A.](#)

327 Let us denote by  $f$  the vector field that defines the system (17):

$$f(x) = \begin{pmatrix} r_1 x_1 \left(1 - \frac{x_1}{K_1}\right) - (m_{12} + qE)x_1 + m_{21} x_2 \\ r_2 x_2 \left(1 - \frac{x_2}{K_2}\right) + m_{12} x_1 - m_{21} x_2 \end{pmatrix}.$$

328 Let  $z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ ,  $y(t) = h(x) = q E x_1(t)$  and

$$329 \Phi(x) = \begin{pmatrix} y \\ \dot{y} \end{pmatrix} = \begin{pmatrix} q E x_1 \\ r_1 q E x_1 \left(1 - \frac{x_1}{K_1}\right) - (m_{12} + qE)q E x_1 + m_{21} q E x_2 \end{pmatrix}.$$

$$330 \text{ Therefore } \frac{d\Phi}{dx} = \begin{pmatrix} qE & 0 \\ r_1 qE - \frac{2r_1 qE x_1}{K_1} - (m_{12} + qE)qE & m_{21} qE \end{pmatrix}$$

331 and  $\text{Det}\left(\frac{d\Phi}{dx}\right) = q^2 E^2 m_{21}$ .

332 As the parameters  $q$ ,  $E$  and  $m_{21}$  are positive ( $\neq 0$ ), we can conclude that  $\text{Det}\left(\frac{d\Phi}{dx}\right) \neq$   
 333  $0$ , and then,  $\Phi$  is a diffeomorphism from  $\mathbb{R}^2$  to  $\Phi(\mathbb{R}^2)$ , thus system (17) is observable.  
 334 Thanks to ([13]) the observer can be expressed as follows:

$$\dot{\hat{x}} = \tilde{f}(\hat{x}) + \left(\frac{d\Phi}{dx}\right)^{-1} \times S(\theta)^{-1} C^T (y - h(\hat{x})), \quad (18)$$

335 where  $\tilde{f}$  is a Lipschitz extension of the function  $f$  from the invariant domain  $D_w$  to  
 336 the whole  $\mathbb{R}^2$  space,  $C = (1, 0)$  and

$$337 S(\theta)^{-1} = \begin{pmatrix} 2\theta & \theta^2 \\ \theta^2 & \theta^3 \end{pmatrix}, \text{ with } \theta \geq 1.$$

338 The restriction of the estimator (18) to the invariant domain  $D_w$  is given by the  
 339 equations:

$$\begin{cases} \dot{\hat{x}}_1 = r_1 \hat{x}_1 \left(1 - \frac{\hat{x}_1}{K_1}\right) - m_{12} \hat{x}_1 + m_{21} \hat{x}_2 - qE \hat{x}_1 + 2\theta(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 = r_2 \hat{x}_2 \left(1 - \frac{\hat{x}_2}{K_2}\right) + m_{12} \hat{x}_1 - m_{21} \hat{x}_2 \\ \quad + 2\theta \left(\frac{qE}{m_{21}} - \frac{1}{m_{21}} + \frac{m_{12}}{qEm_{21}} + \frac{2r_1 x_1}{m_{21}} + \frac{\theta}{m_{21}}\right) (x_1 - \hat{x}_1), \end{cases} \quad (19)$$

### 340 3.2 Simulation

341 Simulations for the model (17) together with its observer (18) have been done with  
 342 the following parameters :

$$343 r_1 = \frac{7}{10}; r_2 = \frac{5}{10},$$

$$344 q = \frac{25}{100}, E = \frac{9}{10},$$

$$345 K_1 = 10, K_2 = \frac{22}{10},$$

$$346 m_{12} = \frac{2}{10}, m_{21} = \frac{1}{10},$$

347 Thanks to formula (20) we compute  $w_0 = 8.987$  and we take  $w = 20$ .

348 With these parameters, the invariant domain is the triangle defined by  $O(0, 0)$ ,  
 349  $A(w, 0) = A(20, 0)$  and  $B(0, w) = B(0, 20)$ , and we take  $\theta = 4$ .

350 Using the SCILAB free software, the time evolution of the states as well as the  
 351 respective estimates when the Lipschitz extension is done are drawn in Figures 8  
 352 and 10. When the Lipschitz extension has not been done, the simulations are given  
 353 in Figures 9 and 11.

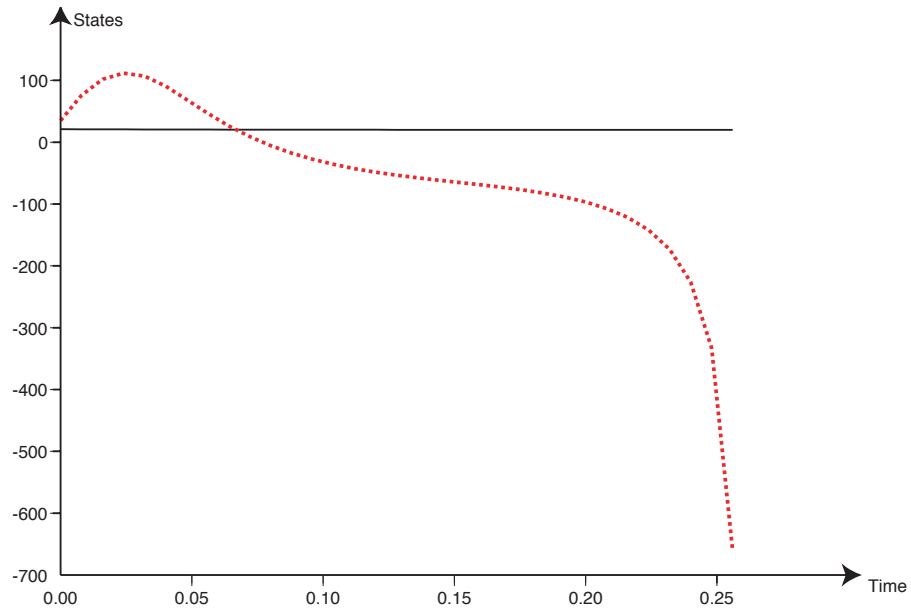


Figure 2: Simulation of system (9) with its observer (15):  $x_0$  (solid line) and its estimate  $\hat{x}_0$  (dashed line) when  $\varphi$  is not extended

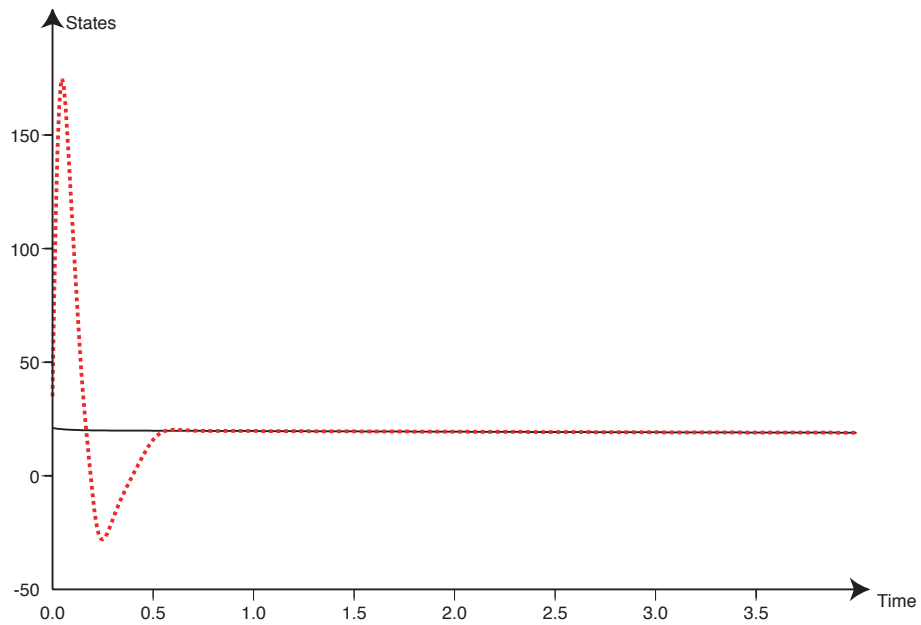


Figure 3: Simulation of system (9) with its observer (15):  $x_0$  (solid line) and its estimate  $\hat{x}_0$  (dashed line) when  $\varphi$  is extended

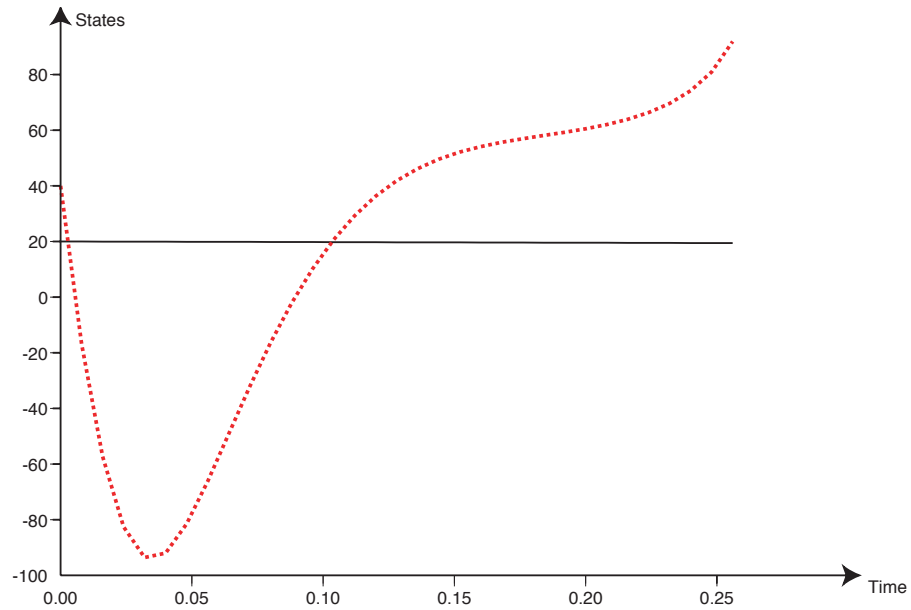


Figure 4: Simulation of system (9) with its observer (15):  $x_1$  (solid line) and its estimate  $\hat{x}_1$  (dashed line) when  $\varphi$  is not extended

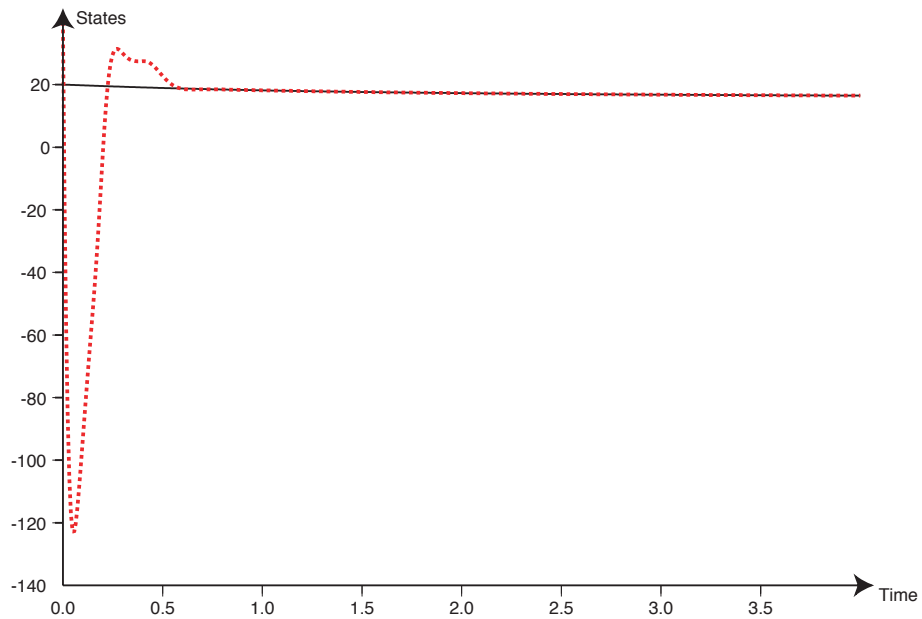


Figure 5: Simulation of system (9) with its observer (15):  $x_1$  (solid line) and its estimate  $\hat{x}_1$  (dashed line) when  $\varphi$  is extended

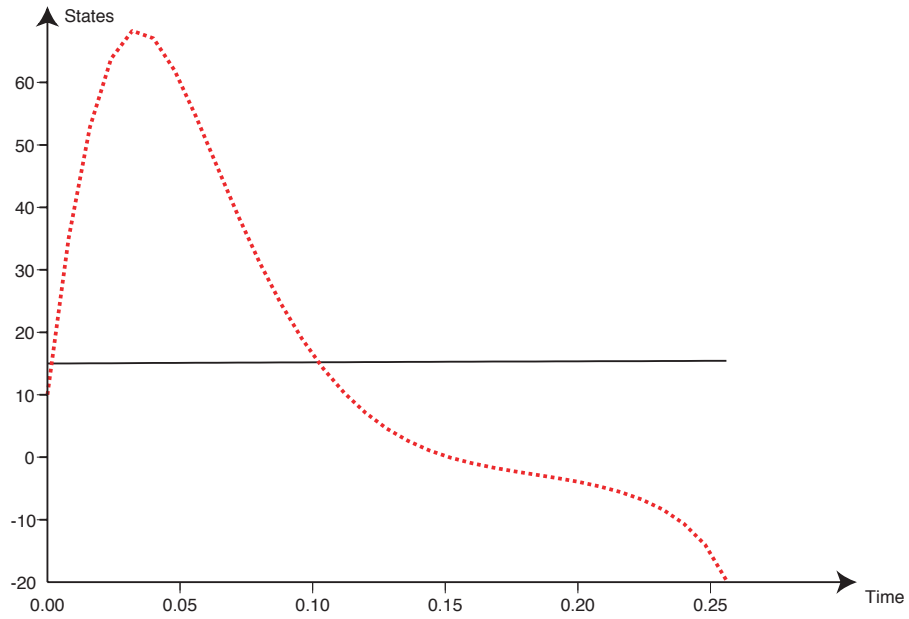


Figure 6: Simulation of system (9) with its observer (15):  $x_2$  (solid line) and its estimate  $\hat{x}_2$  (dashed line) when  $\varphi$  is not extended

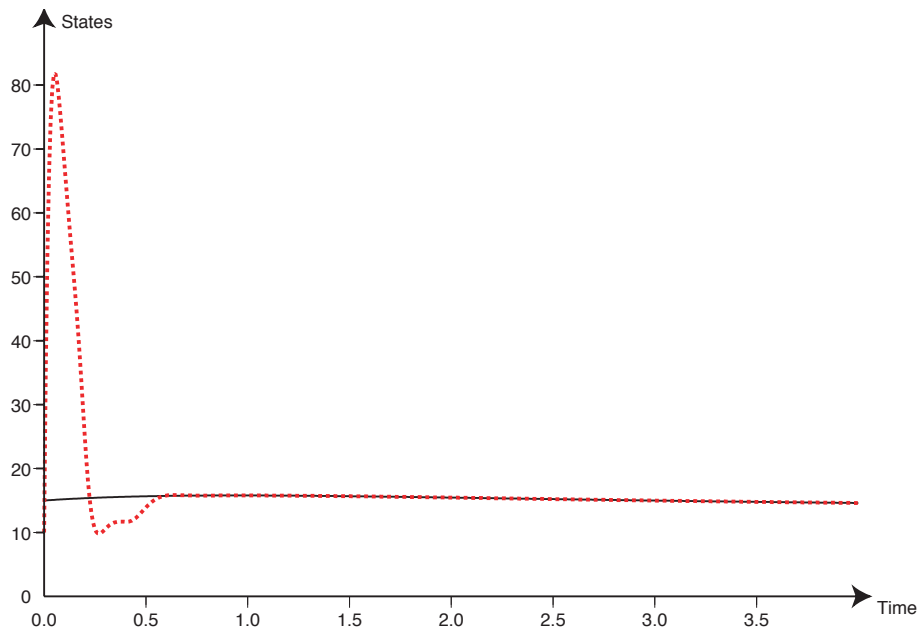


Figure 7: Simulation of system (9) with its observer (15):  $x_2$  (solid line) and its estimate  $\hat{x}_2$  (dashed line) when  $\varphi$  is extended

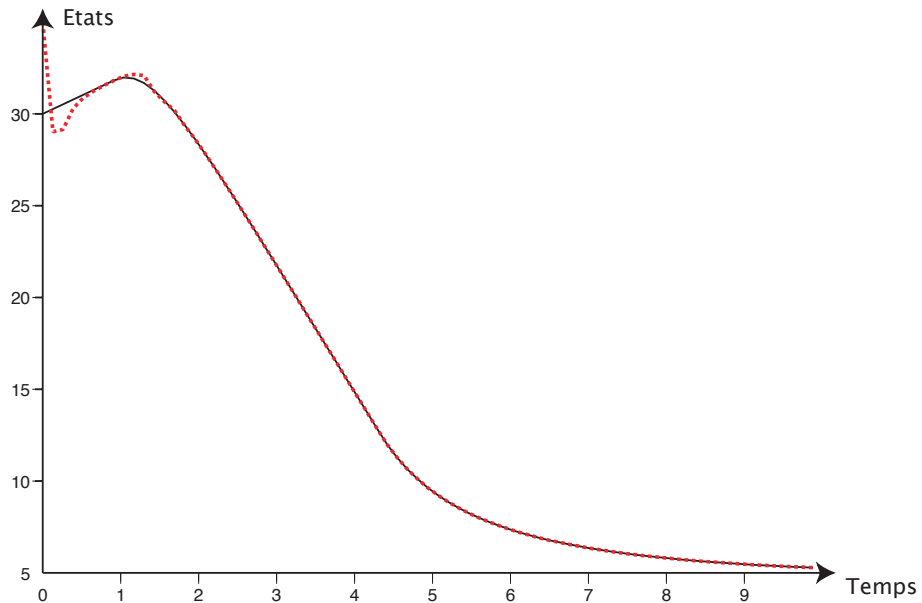


Figure 8: Simulation of system (17) with its observer (18):  $x_1$  (solid line) and its estimate  $\hat{x}_1$  (dashed line) when  $f$  is extended

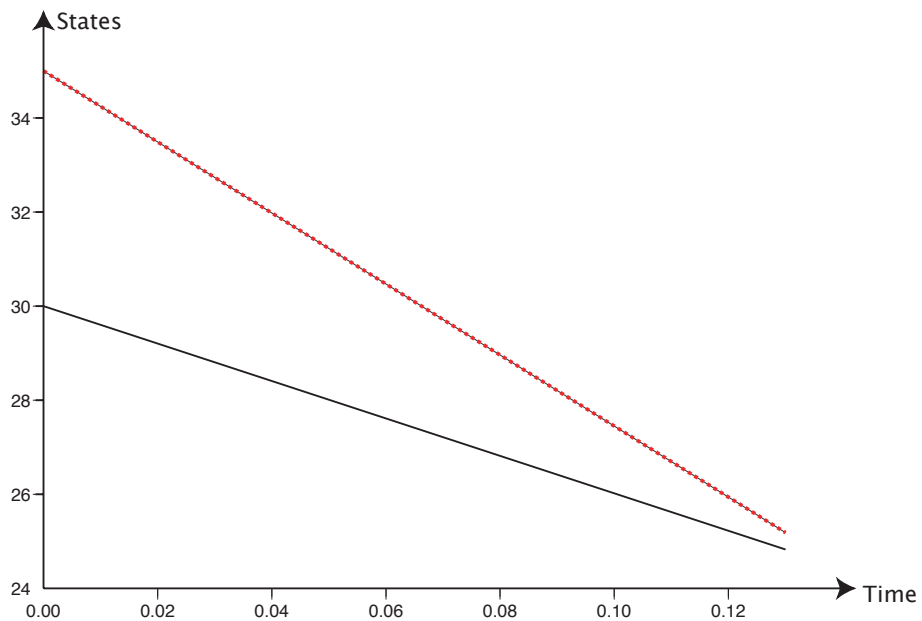


Figure 9: Simulation of system (17) with its observer (18):  $x_1$  (solid line) and its estimate  $\hat{x}_1$  (dashed line) when  $f$  is not extended

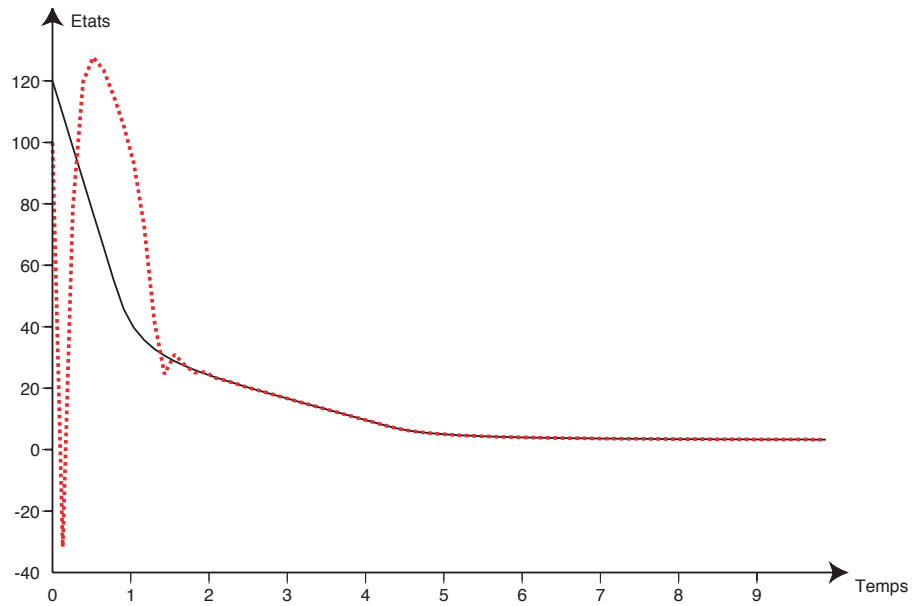


Figure 10: Simulation of system (17) with its observer (18):  $x_2$  (solid line) and its estimate  $\hat{x}_2$  (dashed line) when  $f$  is extended

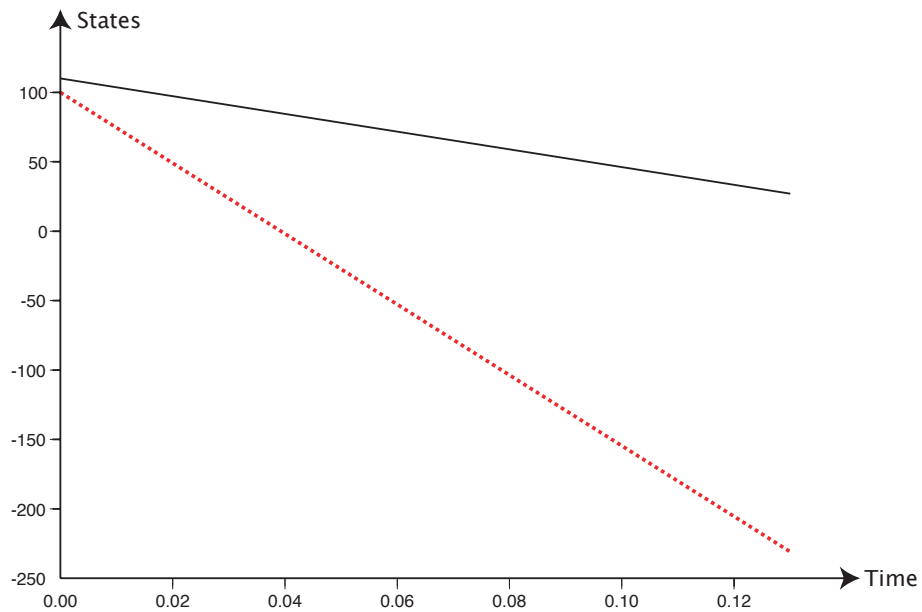


Figure 11: Simulation of system (17) with its observer (18):  $x_2$  (solid line) and its estimate  $\hat{x}_2$  (dashed line) when  $f$  is not extended

## 354 4 Conclusion

355 We have tried to combine modern Control Theory, Computer Science and Mathe-  
356 matics to address the state estimation problem for systems that model the dynamics  
357 of fish populations submitted to a fishing action. Indeed one of the important prob-  
358 lems in fishery sciences is to estimate the state of the resource using the available  
359 data, in order to produce scientific opinions that can be helpful for developing man-  
360 agement policies that need to have a good estimate of the available resource.  
361 In this work, we have constructed High Gain observers for some fishery models.  
362 With the use of judicious value of the gain parameter  $\theta$  we obtain satisfactory es-  
363 timation of the real state. The observer's convergence is quite fast and does not  
364 depend on the initial conditions choice. Therefore one can get a "good" estimate of  
365 the unmeasurable real state very quickly. It is interesting to notice that the state  
366 estimator built in this paper for the stage-structured model use only the total catch  
367 to give not only an estimate of the total stock but also an estimate of the number  
368 of individuals in each stage class. The classical techniques like the Cohort Analysis  
369 (CA) or the Virtual Population Analysis (VPA) use the total catch for each stage-  
370 class in order to give estimates of the number of individuals in each stage class.  
371 In practice it is easier to measure the total catch (without doing any distinction  
372 between individuals) than to measure the catch for each stage class. However the  
373 observers given in this paper assume that the model is good enough and that the  
374 parameters values are available.  
375 Nonlinear control techniques are useful for studying and controlling complex systems.  
376 Although they have been initially developed for mechanical and electrical systems  
377 their applications to biological and environmental problems are growing. Tools of  
378 optimal control theory have been extensively used in renewable resource management  
379 ([8], [3] [23] [18] [25],[31],[45],[12],[32]). The present paper shows that the estimation  
380 problem in fisheries management can also be investigated from the point of view of  
381 control engineering.

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## 497 Appendix A. Positive invariance of $D_w$

498 Let  $N = x_1 + x_2$ .

$$499 \dot{N} = -qEx_1 + r_1 \left(1 - \frac{x_1}{K_1}\right) x_1 + (N - x_1) \left(1 - \frac{N - x_1}{K_2}\right) r_2$$

500 Let  $w$  be a positive real number, for  $N = w$ , we have

$$501 \dot{N} = -qEx_1 + r_1 \left(1 - \frac{x_1}{K_1}\right) x_1 + (w - x_1) \left(1 - \frac{w - x_1}{K_2}\right) r_2 = g(x_1)$$

502 The function  $g$  is defined for  $0 \leq x_1 \leq w$ .

$$503 g(0) = w \left(1 - \frac{w}{K_2}\right) r_2$$

$$504 g(w) = -quw + w \left(1 - \frac{w}{K_1}\right) r_1$$

$$505 g'(x_1) = r_1 - r_2 - qu + \frac{2wr_2}{K_2} - 2 \left(\frac{r_1}{K_1} + \frac{r_2}{K_2}\right) x_1$$

$$506 g'(x_1) = 0 \Leftrightarrow x_1 = \bar{x}_1 = \frac{K_1(K_2r_1 - K_2r_2 - quK_2 + 2wr_2)}{2(K_2r_1 + K_1r_2)}$$

507 The maximum value of the function  $g$  is then given by the expression

$$508 \frac{K_1K_2(qu - r_1 + r_2)^2 + (4K_2r_1r_2 + K_1(-4qur_2 + 4r_1r_2))w - 4(r_1r_2)w^2}{4(K_2r_1 + K_1r_2)}$$

509 It is therefore clear that this maximum is non positive if  $w \geq w_0$  with

$$w_0 = \frac{r_1r_2(K_1 + K_2) - quK_1r_2 + \sqrt{r_2(K_2r_1 + K_1r_2)(K_1(-qu + r_1)^2 + K_2r_1r_2)}}{2r_1r_2} \quad (20)$$

510 This shows that for any real number  $w \geq w_0$ , the compact set

$$D_w = \{(x_1, x_2) \in \mathbb{R}_+^2 : x_1 + x_2 \leq w\}$$

511 is positively invariant for system (17).

## 512 Appendix B. Construction of the Lipschitz extension of $\varphi$

514 The function  $\varphi$  is Lipschitz on the compact set  $D = [a_0, b_0] \times [a_1, b_1] \times [a_2, b_2]$ . Our aim  
515 is to extend it to a function  $\tilde{\varphi}$  which is Lipschitz with the same Lipschitz coefficient  
516 in the whole  $\mathbb{R}^3$ .

517 Let  $a(a_0, a_1, a_2)$ , (respectively  $b(b_0, b_1, b_2)$ ), the lower corner, (respectively the upper  
518 corner) of the domain  $D$  and  $x(x_0, x_1, x_2)$  an unspecified point of  $\mathbb{R}^3$ .

519 The problem of the extension is set for point  $x \notin D$ ; in this situation we have 26  
520 possibilities according to the situation of  $x$ . The different situations correspond to  
521  $x_i \leq a_i$ ,  $a_i \leq x_i \leq b_i$ , or  $x_i \geq b_i$ .

522 The principle of this prolongation is to compose the function  $\varphi$  with the function  $\pi$   
523 (the projection function of the point  $x$  on the domain  $D$ ).

524 The extension of function  $\varphi$  is described by the following algorithm:

525 if  $x_0 \leq a_0$  then

526 if  $x_1 \leq a_1$  then

```

527     if  $x_2 \leq a_2$  then
528          $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, a_1, a_2)$ 
529     else
530         if  $x_2 \leq b_2$  then
531              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, a_1, x_2)$ 
532         else
533              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, a_1, b_2)$ 
534         end.
535     end.
536 else
537     if  $x_1 \leq b_1$  then
538         if  $x_2 \leq a_2$  then
539              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, x_1, a_2)$ 
540         else
541             if  $x_2 \leq b_2$  then
542                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, x_1, x_2)$ 
543             else
544                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, x_1, b_2)$ 
545             end.
546         end.
547     else
548         if  $x_2 \leq a_2$  then
549              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, b_1, a_2)$ 
550         else
551             if  $x_2 \leq b_2$  then
552                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, b_1, x_2)$ 
553             else
554                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(a_0, b_1, b_2)$ 
555             end.
556         end.
557     end.
558 end.
559 else
560     if  $x_0 \leq b_0$  then
561         if  $x_1 \leq a_1$  then
562             if  $x_2 \leq a_2$  then
563                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, a_1, a_2)$ 
564             else
565                 if  $x_2 \leq b_2$  then
566                      $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, a_1, x_2)$ 
567                 else
568                      $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, a_1, b_2)$ 
569                 end.
570             end.
571         else
572             if  $x_1 \leq b_1$  then

```

```

573         if  $x_2 \leq a_2$  then
574              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, x_1, a_2)$ 
575         else
576             if  $x_2 \leq b_2$  then
577                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, x_1, x_2)$ 
578             else
579                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, x_1, b_2)$ 
580             end.
581         end.
582     else
583         si  $x_2 \leq b_2$  then
584              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, b_1, x_2)$ 
585         else
586              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(x_0, b_1, b_2)$ 
587         end.
588     end.
589 end.
590 else
591     if  $x_1 \leq a_1$  then
592         if  $x_2 \leq a_2$  then
593              $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, a_1, a_2)$ 
594         else
595             if  $x_2 \leq b_2$  then
596                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, a_1, x_2)$ 
597             else
598                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, a_1, b_2)$ 
599             end.
600         end.
601     else
602         if  $x_1 \leq b_1$  then
603             if  $x_2 \leq a_2$  then
604                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, x_1, a_2)$ 
605             else
606                 if  $x_2 \leq b_2$  then
607                      $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, x_1, x_2)$ 
608                 else
609                      $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, x_1, b_2)$ 
610                 end.
611             end.
612         else
613             if  $x_2 \leq a_2$  then
614                  $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, b_1, a_2)$ 
615             else
616                 if  $x_2 \leq b_2$  then
617                      $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, b_1, x_2)$ 
618                 else

```

```
619          $\tilde{\varphi}(x_0, x_1, x_2) = \varphi(b_0, b_1, b_2)$ 
620     end.
621 end.
622 end.
623 end.
624 end.
625 end.
```